

COMPLEX ANALYSIS II - PROBLEMS

PROBLEMS FOR DISCUSSION

- 1** Find the maximum modulus function for the entire functions:

$$f_1(z) = e^z, \quad f_2(z) = e^{e^z}, \quad f_3(z) = \cos(z), \quad f_4(z) = e^{\cos(z)}.$$

- 2** Estimate the maximum modulus function for $f(z) = \sin(z)$.

- 3** Let $f(z) = \cos(az^2)$, where $a \in \mathbb{C}^*$. Find $\rho(f)$ and $\tau(f)$.

- 4** Let f be a transcendental entire function of order $\rho(f) \in]0, +\infty[$ and $P(z) \neq 0, Q(z)$ be two polynomials of degree n et m respectively. Write a proof for the following equality:

$$\tau(Pf + Q) = \tau(f).$$

- 5** Determine whether the following functions are entire, and determine their order and type (when it's defined):

$$f(z) = \sum_{n=0}^{+\infty} \frac{A^n z^n}{\Gamma(1 + \alpha n)} \quad (A > 0, \alpha > 0); \quad g(z) = \int_0^1 e^{zt^2} dt.$$

SUPPLEMENTARY PROBLEMS

- 1** For an entire function f , show that the maximum modulus function $M(r, f)$ is a Lipschitz function on every compact of \mathbb{R}_+ .

- 2** Let f be a transcendental entire function, and let $\lambda \in]0, 1[$. Show that

$$\lim_{r \rightarrow +\infty} \frac{M(\lambda r, f)}{M(r, f)} = 0$$

- 3** Let f and g be two entire functions of orders, respectively, $\rho_f, \rho_g \in]0, +\infty[$. Prove the following statement:

- (a) If $\rho_f = \rho_g$, then

$$\tau(f + g) \leq \max\{\tau(f), \tau(g)\},$$

$$\tau(fg) \leq \max\{\tau(f), \tau(g)\}.$$

- (b) If $\rho_f > \rho_g$ or $\tau(f) > \tau(g)$, then $\tau(f + g) = \tau(f)$.

- (c) If $\rho_f = \rho_g$ or $\tau(f) > \tau(g)$, then $\rho(f + g) = \rho_f = \rho_g$.

- 4** Determine whether the following function is entire, and determine its order and type (when it's defined):

$$f(z) = \int_0^{+\infty} e^{-t^\alpha} \cos(zt) dt \quad (\alpha > 1).$$